An Introduction to Computational Finance

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The aim of this course is to provide an introduction to quantitative techniques to price financial instruments. The course will offer an overview of the main theoretical elements of applied financial models. It will develop the most relevant numerical methods to price financial derivatives. It will put those methods into practice by surveying a number of applications and by extending its implementation to evaluate other financial instruments such as corporate liabilities. In particular, we will study two different models (the binomial model and a more general asset pricing model) and the respective most used computational tools (the binomial tree method and the Monte Carlo method). We will provide essential links to academic work, and we will consider a wide range of practical problems: these will include hedging strategies (Greeks and stop-loss); option pricing, with reference to both vanilla style (European and American) and exotic style (Asian, Bermudan, Barrier, Exchange) options; and corporate debt evaluation, with a particular focus on the risk structure of interest rates.

Prerequisites The course is aimed at practitioners or researchers working for Investment Banks, Central Banks or Research Centers/Universities. Participants should have a knowledge of Calculus, Probability, Finance and Economics at a Master or first year PhD level. In addition, participants should have a basic knowledge of programming in MATLAB.

Course outline

The course is taught in 4 sessions of 4 hours each. The material will be self-contained.

Lecture 1. The Binomial Model: basic theoretical elements of arbitrage pricing.

This lecture will introduce the binomial model. Although it represents the most elementary financial model, it helps in understanding virtually all the key theoretical concepts in finance in a simple environment. Importantly, its applications are still widely used by practitioners. We will study the notions of arbitrage and risk neutral valuation in a one-period, two-state environment, then we will extend our analysis to a multi-period model.

Readings

- Björk T. (2009), *Arbitrage Theory in Continuous Time*, Oxford University Press. Ch 1-2.
- Cox J., Ross S., and Rubinstein M. (1979), "Option Pricing: A Simplified Approach," *Journal of Financial Economics*, 7, 229–264.
- Cox J. and Rubinstein M. (1992), *Options Markets*, Prentice Hall. Ch 5.
- Rendleman R. and Bartter B. (1979), "Two State Option Pricing," *Journal of Finance*, 34, 1092–1110.

Lecture 2. Numerical methods: Binomial tree.

This lecture will illustrate the binomial tree method to price financial derivatives. This method relies on a discretization of the underlying stochastic process. To obtain a practical pricing procedure, we calibrate the tree so that it reflects the underlying model, which naturally is a continuous-time, continuous-state stochastic differential equation. We discuss several applications of this approach, which offers the possibility to build a comparison between the different techniques. Particular attention is however paid to implement American option pricing, since the Monte Carlo method is unsuitable to evaluate this type of financial derivatives.

Readings

- Brandimarte P. (2006), *Numerical Methods in Finance and Economics*, Wiley. Ch 2&7.
- Clewlow L. and Strickland C. (1998), *Implementing Derivatives Models*, Wiley. Ch 2.
- Higham D. J. (2002), "Nine Ways to Implement the Binomial Method for Option Valuation in MATLAB," *SIAM Review*, 44, 661-677.
- Levy G. (2004), *Computational Finance. Numerical Methods for Pricing Financial Instruments*, Elsevier, Ch 10&12.

Lecture 3. A More General Framework: arbitrage pricing with infinite states and time periods.

This lecture will generalize the concepts previously developed to the case of an infinite number of states and periods of time. We will overview the notions of Wiener process, stochastic calculus, Itô's lemma, Brownian motion, stochastic and partial differential equations, which ultimately represent theoretical elements instrumental to pricing financial derivatives through the Black-Scholes equation. We will discuss the concepts of completeness and hedging, and define the sensitivity measures known as the Greeks. Furthermore, we will review the theoretical link between pricing financial derivatives and evaluating corporate liabilities, and use the resulting mathematical tool to analyze the risk structure of corporate debt.

Readings

- Björk T. (2009), *Arbitrage Theory in Continuous Time*, Oxford University Press. Ch 3-9.
- Black F. and Scholes M. (1973), "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81, 659–683.
- Merton R. C. (1969), "Lifetime Portfolio Selection under Uncertainty: the Continuous-Time Case," *Review of Economics and Statistics*, 51, 247-257.
- Merton R. C. (1974), "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 29, 449-470.
- Øksendal B. (2007), *Stochastic Differential Equations: An Introduction with Applications*, Springer. Ch 11.

Lecture 4. Numerical methods: Monte Carlo.

This lecture will describe the Monte Carlo method to price financial derivatives and to evaluate corporate liabilities. This method is a key tool in computational finance: it is widely used to evaluate portfolio management rules, to price options, to simulate hedging strategies, and to estimate value at risk. It is generally easy to implement and quite flexible, though it may incur in heavy computational burden. We will illustrate the application of this technique to a number of examples, including some path-dependent options. Then, we will compare the results of our simulations with those obtained by the binomial tree method and, whenever possible, by analytical formulas.

Readings

- Boyle P. (1977), "Options: A Monte Carlo Approach," *Journal of Financial Economics*, 4, 323-338.
- Boyle P., Broadie M., and Glasserman P. (1997), "Monte Carlo Methods for Security Pricing," *Journal of Economics Dynamics and Control*, 21, 1267-1321.
- Brandimarte P. (2006), *Numerical Methods in Finance and Economics*, Wiley. Ch 4&8.
- Glasserman P. (2004), *Monte Carlo Methods in Financial Engineering*, Springer.
- Jaeckel P. (2002), *Monte Carlo Methods in Finance*, Wiley.