Numerical Methods for Time Inconsistency, Private Information and Limited Commitment

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Course Description: Many dynamic economic models have a **recursive** structure, i.e. the equilibrium or the optimal allocation can be represented as a function of the state variables. Dynamic programming techniques are well suited to numerically solve these models: all the variables are functions of something that happened or was chosen in the past. However, there is a large class of economic models in which expectations of future choices affect how current choices are taken. Hence, they are not recursive in the standard sense (i.e., in these models the past states are not a sufficient statistic for determining current choices). Examples are problems of optimal policy with a benevolent government suffering from time inconsistency (very common in macroeconomics), and dynamic contracts with limited commitment and private information environments (quite common in industrial organization and game theory). The literature has shown two possible ways for "recursifying" these problems, which exploit the idea of "creating" auxiliary state variables which makes the problem recursive in the standard sense. The first method uses agent's future lifetime utilities (aka **promised values** or continuation values) as state variables. The second technique uses (functions of) past Lagrange multipliers as state variables, and it is therefore called **the Lagrangian approach**. Both these techniques then use dynamic programming arguments to show that a recursive solution exists in the newly created state space. This is a crash course on computational techniques that help solving those models. We will cover a bit of theory, some basic numerical tools, dynamic programming and projection methods, and the two major approaches for these models: the promised utilities approach and the Lagrangian approach. Teaching method is hands-on: we will look at concrete economic applications, and numerical techniques are presented with Matlab codes.

This course is for: Economists working on game theory, industrial organization, macroeconomic theory, labour economics, mechanism design.

Software: Matlab, GitHub (for distributing material and for future support)
Prerequisites: Knowledge of Matlab programming language is required. Main concepts of real analysis (sequences, correspondences, etc.) are taken as given.

Readings: Lectures notes are provided. Numerical techniques are mostly based on Judd (1998) and Miranda and Fackler (2002). Additional readings are included in the outline.

Course Outline:

Each lecture is 4 hours.

Lecture 1

- We will explain the basic concepts of recursivity in economics, and have a look at dynamic programming theory. We will then move to computational methods, and write code for performing value function iteration and policy function iteration, in a simple application to a standard economic model.
- Material: Lecture notes, Matlab codes provided, Ljungqvist and Sargent (2012), ch. 3-4, Stokey and Lucas (1989), chs. 1-2 and 4, Adda and Cooper (2003), ch. 2-3

Lecture 2

- We will provide a few examples of non-recursivity in economics, related to time inconsistency, private information and limited commitment. Before analysing in depth the two approaches to recursify them, we will first look at numerical techniques (integration, differentiation, non-linear solvers, etc.) needed to compute solutions of these models, and will spend most of the day on projection methods. We will then look at a simple (recursive) example solved with the orthogonal collocation method.
- Material: Lecture notes, Matlab codes provided, Judd (1998), ch. 11, several chapters from Miranda and Fackler (2002) which will be indicated in class

Lecture 3

- We will first cover the promised utilities approach. This technique recursifies the problem by using continuation values as new state variables. However, a complication that derives from this approach is that those continuation values are constrained to belong to a feasible set which needs to be calculated endogenously. We will look at the theory following Abreu, Pearce, and Stacchetti (1990), and then provide a numerical method to characterize the feasible set, following Judd, Yeltekin, and Conklin (2003).
- Material: Lecture notes, Matlab codes provided, Ljungqvist and Sargent (2012), ch. 16, 20-22, Abreu, Pearce, and Stacchetti (1990), Judd, Yeltekin, and Conklin (2003)

Lecture 4

- Finally, we will analyse the Lagrangian approach. This is a method that uses functions of past Lagrange multipliers as new state variables, in order to recursify the problem. The main advantage is that Lagrange multipliers' feasible set does not have to be computed. We will first examine the theoretical aspects, and then we will look at a numerical example with private information.

- Material: Lecture notes, Matlab codes provided, Ljungqvist and Sargent (2012), ch. 16, 20-22, Marcet and Marimon (2011), Mele (2014)

References

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Marcet, A., and R. Marimon (2011): "Recursive Contracts," Discussion paper.

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Miranda, M. J., and P. L. Fackler (2002): *Applied Computational Economics and Finance*. MIT Press, Cambridge, MA, USA.

Stokey, N. L., and R. E. Lucas (1989): *Recursive Methods in Economic Dynamics*. Harvard University Press.